Standing Wave Questions
Question 1

The picture shows an Irish harp – an instrument that is played by plucking the strings. One of the harp strings is 43.2 cm long.

(a) Calculate the wavelength of the fundamental note played on this string when it is plucked.
(b) On the line below sketch the 3rd overtone (4th harmonic) on this string.
Another harp string is 57.8 cm long and has a mass of $4.64 \times 10^{-4}$ kg. The tension force in the string is 70 N. The wave speed in this string can be shown using the formula:

$$v = \sqrt{\frac{T}{\mu}}$$

Where $T$ is the tension force and $\mu$ is the mass per unit length on the string.

(c) By finding the mass per unit length, show that the wave speed is 296 m.s$^{-1}$.

Find mass per unit length, $\mu = 7.993 \times 10^{-4}$ kg.m$^{-1}$.

$$v = \sqrt{\frac{70}{7.993 \times 10^{-4}}}$$

$$v = 295.933 \text{m.s}^{-1}$$

The tension force on the harp string is now increased.

(d) State what happens to the size of the wavelength AND frequency of the wave on the string.

When the string is tighter:
- Increased tension = increased frequency (because velocity has increased)
- However, wavelength remains the same (the wave is fixed at each end)
(e) During a concert a flute is heard playing alongside the harp. Explain why the same note played on a flute sounds different to that played on the harp.

While the note may be the same:

The instruments produce different overtones and different strengths of overtones

What you hear is a combination of the fundamental frequency and overtones. These waves overlap (interfere) to produce the resultant wave.

Therefore, the resultant wave produces a different sound for each instrument.
Question 1 continued

Speed of sound in the air is $3.4 \times 10^2$ m.s$^{-1}$.

(f) A particular flute can be modeled as an open pipe of length 0.61 m. Calculate the lowest possible frequency note that could be played on this flute.

Lowest frequency has the longest wavelength (fundamental frequency).

For an open pipe:

$$f = \frac{v}{\lambda}$$

$$v = f \lambda$$

Length = $\frac{1}{2} \lambda = 0.61$ m

$$f = \frac{3.4 \times 10^2}{1.22}$$

$$f = 278.69 \text{Hz} = 280 \text{Hz}$$
Standing Wave Notes

\( \nu \) can be altered with:

1. Different medium (e.g. heavier rope)
2. Different tension

\( f \) relates to pitch:
- lower \( \nu = \) lower \( f \)
  (=So lower pitch)

velocity is set
\[ \nu = f \lambda \]

Resonant frequency

Number of half (or quarter) wavelengths

There are 3 wavelengths
(6 half wavelengths)
Standing Wave Types

Because there is a node at each end, there must be a half number of wavelengths.
Standing Wave Types

Because there is a node one end, it starts at \( \frac{1}{4} \) wavelength then goes up by \( \frac{1}{2} \) wavelength.
Standing Wave Types

Because there is an antinode at each end, there must be a half number of wavelengths.
A child’s toy consists of a long, flexible, plastic pipe, open at both ends. Holding the pipe at one end, the other end can be swung around so that a standing wave is set up in the pipe, and a musical note heard. Explain how a standing wave is set up in the pipe.

Waves travel along the pipe and are reflected back from the open end to create standing waves.

Because it is an open pipe, there are antinodes at each end.

Only a certain number of wavelengths can fit into the pipe. The flow of air through the pipe causes waves to be generated. Only the (half) wavelengths that ‘fit’ with the pipe and resonate to create a standing wave.
Standing Wave Harmonics

Opening the holes of a wind instrument creates an **ANTINODE** at those points

- **Mouthpiece**
- **Node**
- **Antinode**

**a "low note"**

**First hole open**

**Antinode**

**a "high" note**

**First hole open**

**Antinode**

**Node**

**Antinode**
Recall Pipe Types

- Open pipe
- Closed pipe
- String
Overall: Recall That

**Fundamental frequency**

*The lowest frequency produced in any standing wave*

**Overtone**

*A frequency higher than the fundamental*

**Harmonics**

*A multiple of the fundamental frequency*

Remember these are standing waves
String

Fundamental
1st Harmonic

First Overtone
2nd Harmonic

Second Overtone
3rd Harmonic

Third Overtone
4th Harmonic

And so on...
Open Pipes

Fundamental frequency
½ wavelength

Overtone
Number: start counting after the fundamental

Harmonics
Number: start counting at the fundamental
Closed Pipe

Fundamental frequency

1/4 wave

Overtone

Number: start counting after the fundamental

Harmonics

Number: count the number of quarters

Remember, harmonics are a multiple of the fundamental frequency
Calculations

If the pipe/string below is 0.4m. Calculate the wavelength of each and name the wave.
Open Pipe Calculations

0.5λ = 0.4m  λ = 0.8m

Fundamental 1st Harmonic

1λ = 0.4m  λ = 0.4m

1st Overtone 2nd Harmonic

1.5λ = 0.4m  λ = 0.27m

2nd Overtone 3rd Harmonic
String

0.5\(\lambda = 0.4m\) \(\lambda = 0.8m\) \(\text{Fundamental 1}\text{st Harmonic}\)

1.5\(\lambda = 0.4m\) \(\lambda = 0.27m\) \(\text{2}\text{nd Overtone 3}\text{rd Harmonic}\)

2.5\(\lambda = 0.4m\) \(\lambda = 0.16m\) \(\text{4}\text{th Overtone 5}\text{th Harmonic}\)
Closed Pipe

0.25\(\lambda\) = 0.4\(m\) \quad \lambda = 1.6\(m\) \quad \text{Fundamental 1}\(^{st}\) Harmonic

0.75\(\lambda\) = 0.4\(m\) \quad \lambda = 0.53\(m\) \quad \text{1}\(^{st}\) Overtone 3\(^{rd}\) Harmonic

1.25\(\lambda\) = 0.4\(m\) \quad \lambda = 0.32\(m\) \quad \text{2}\(^{nd}\) Overtone 5\(^{th}\) Harmonic
Standing Wave Harmonics

In an instrument, many harmonics/overtones can exist at the same time.

When we hear a note from an instrument:

- The sound is made up of a combination of the harmonics/overtones.
Standing Wave Harmonics

All of these harmonics/overtones can exist at the same time

*When we hear a note from an instrument:*

The sound is made up of a combination of the harmonics/overtones

Different instruments produce different overtones at different strengths

These interfere, making in a different resultant wave, which sounds different
In Summary

<table>
<thead>
<tr>
<th>Wavelength, $\lambda$</th>
<th>Frequency, $f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1 = 2L$</td>
<td>$f_1 = \nu / \lambda_1$</td>
</tr>
<tr>
<td>$\lambda_2 = L$</td>
<td>$f_2 = 2f_1$</td>
</tr>
<tr>
<td>$\lambda_3 = 2L/3$</td>
<td>$f_3 = 3f_1$</td>
</tr>
<tr>
<td>$\lambda_4 = 2L/4$ or $\frac{1}{2}L$</td>
<td>$f_4 = 4f_1$</td>
</tr>
<tr>
<td>$\lambda_n = 2L/n$</td>
<td>$f_n = n \nu / 2L$ for $n = 1, 2, 3, \ldots$</td>
</tr>
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Fundamental Frequency or 1st Harmonic
2nd Harmonic
3rd Harmonic
4th Harmonic
## Modes of vibration for an open tube

<table>
<thead>
<tr>
<th>Picture of Standing Wave</th>
<th>Name</th>
<th>Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Standing Wave" /></td>
<td><strong>1st Harmonic</strong></td>
<td>2 Antinodes, 1 Node</td>
</tr>
<tr>
<td><img src="image" alt="Standing Wave" /></td>
<td><strong>2nd Harmonic</strong></td>
<td>3 Antinodes, 2 Nodes</td>
</tr>
<tr>
<td><img src="image" alt="Standing Wave" /></td>
<td><strong>3rd Harmonic</strong></td>
<td>4 Antinodes, 3 Nodes</td>
</tr>
<tr>
<td><img src="image" alt="Standing Wave" /></td>
<td><strong>4th Harmonic</strong></td>
<td>5 Antinodes, 4 Nodes</td>
</tr>
<tr>
<td><img src="image" alt="Standing Wave" /></td>
<td><strong>5th Harmonic</strong></td>
<td>6 Antinodes, 5 Nodes</td>
</tr>
</tbody>
</table>

- $L = \frac{1}{2} \lambda_1$
- $f_1 = \frac{v}{2L}$

- $L = \lambda_2$
- $f_2 = \frac{v}{L}$

- $L = 1\frac{1}{2} \lambda_3$
- $f_3 = 3\frac{v}{2L}$

- $L = 2\lambda_4$
- $f_4 = 2\frac{v}{L}$

- $L = 2\frac{1}{2} \lambda_5$
- $f_5 = 5\frac{v}{2L}$
Modes of vibration for a closed tube

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<tr>
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<tbody>
<tr>
<td><img src="image1" alt="Standing Wave" /></td>
<td>1st Harmonic or Fundamental</td>
<td>1 Antinode 1 Node</td>
</tr>
<tr>
<td><img src="image2" alt="Standing Wave" /></td>
<td>3rd Harmonic or 1st Overtone</td>
<td>2 Antinodes 2 Nodes</td>
</tr>
<tr>
<td><img src="image3" alt="Standing Wave" /></td>
<td>5th Harmonic or 2nd Overtone</td>
<td>3 Antinodes 3 Nodes</td>
</tr>
<tr>
<td><img src="image4" alt="Standing Wave" /></td>
<td>7th Harmonic or 3rd Overtone</td>
<td>4 Antinodes 4 Nodes</td>
</tr>
<tr>
<td><img src="image5" alt="Standing Wave" /></td>
<td>9th Harmonic or 4th Overtone</td>
<td>5 Antinodes 5 Nodes</td>
</tr>
</tbody>
</table>

L = \( \frac{1}{4} \lambda_1 \)

\( f_1 = \frac{v}{4L} \)

L = \( \frac{3}{4} \lambda_3 \)

\( f_3 = \frac{3v}{4L} \)

L = \( 1\frac{1}{4} \lambda_5 \)

\( f_5 = \frac{5v}{4L} \)

L = \( 1\frac{3}{4} \lambda_7 \)

\( f_7 = \frac{7v}{4L} \)

L = \( 2\frac{1}{4} \lambda_9 \)

\( f_9 = \frac{9v}{4L} \)
Example 1

- A shower acts like a closed pipe with a node at both ends. Sam’s shower has a height of 2.40 m, with a square base of width 1.10 m. The diagram shows a side view of the shower with one of the standing sound waves that can be set up in the shower. The displacement antinode (A) and nodes (N) are shown on the diagram.

Speed of sound in air = $3.43 \times 10^2 \text{ m s}^{-1}$
Speed of sound in water = $1.49 \times 10^3 \text{ m s}^{-1}$

(a) Show that the frequency of the vertical standing sound wave drawn is 71.5 Hz.

$$\lambda = 2L = 4.80 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{343}{4.80}$$

$$f = 71.5 \text{ Hz}$$
Example 1 continued

- Sam loves singing in the shower. Although Sam is a talented singer he cannot sing a note to resonate at this low a frequency. However, Sam can produce two resonant frequencies: • a vertical standing wave at 143 Hz • a horizontal standing wave at 156 Hz.
- Draw these two standing waves in the box on the right. Show the calculations you used, in order to draw the two waves.

Vertically: \( \lambda = \frac{v}{f} = \frac{343}{143} = 2.40 \text{ m} \) so one \( \lambda \) would have a fundamental note of 143 Hz.

Horizontally: \( \lambda = \frac{v}{f} = \frac{343}{156} = 2.20 \text{ m} \) so half \( \lambda \) would have a fundamental note of 156 Hz.
Example 1 continued

• One day, Sam finds his shower is filling with water because the shower waste pipe is blocked. He drains water from the waste pipe, and attempts to locate the position of the blockage. With a loudspeaker, Sam detects the fundamental frequency, and then detects the next two adjacent resonant frequencies at $1.80 \times 10^2$ and $3.00 \times 10^2$ Hz. He uses these resonant frequencies to estimate that the pipe is blocked 1.43 m from the open end.

• Show how Sam calculated that the pipe is blocked 1.43 m from the open end.

$$f_3 = 180 \text{ Hz and } f_5 = 300 \text{ Hz so the fundamental would be } 60 \text{ Hz.}$$

$$f_n = n f_1$$

$$v = f \lambda \quad \lambda = \frac{v}{f} = \frac{343}{60} = 5.72 \text{ m}$$

The fundamental is $4L$ so the blockage is 1.43m from the open end.

$$f_3 = 180 \text{ Hz, } \lambda = \frac{343}{180} = 1.91 \text{ m, } L = 3 \times \frac{1.91}{4} = 1.43 \text{ m}$$

$$f_5 = 300 \text{ Hz, } \lambda = \frac{343}{300} \frac{343}{300} = 1.14 \text{ m, } L = 5 \times \frac{1.14}{4} = 1.43 \text{ m.}$$
Example 2

- Some police forces have used whistles that have two chambers of different lengths.
- A model of the whistle chamber is shown in the diagram below.

(a) On the above diagram, draw the fundamental standing wave in the shorter chamber, AND label any displacement nodes and antinodes.
Example 2 continued

- The fundamental frequencies for the two chambers are 2136 Hz and 1904 Hz. The speed of sound in air is 343 m s–1.
- (b) Calculate the length of the longer chamber.

\[
\lambda = \frac{v}{f}
\]

\[
343 \div 1904 = 0.18 \text{ m.}
\]

\[
0.18 \div 4 = 0.045 \text{ m}
\]

- (c) Explain how a standing wave is produced in a pipe that is closed at one end.

Sound waves enter at the open end, travel along the pipe and reflect from the closed end. Reflected waves are out of phase making the closed end a place of permanent destructive interference (a node). Reflected waves of the correct wavelength reflect from the open end in phase with incident waves, producing a position of permanent constructive interference (an antinode). Amplitude at antinode is larger than amplitude of the wave.
Example 2 continued

- (d) When the whistle is blown, the sound made is quite different to a pure sound of either 2136 Hz or 1904 Hz.
- Calculate the value of TWO other frequencies produced, AND explain why these other frequencies are produced, and what effect they have on the sound.

Overtones in shorter chamber produced at 6408Hz, 10680Hz. Overtones in longer chamber produced at 5712Hz, 9520Hz. Mixture of overtones produces distinctive sound (timbre) of the whistle. Difference tones/beats can be produced between fundamental frequencies.
Example 3

- The Sea Organ in Zadar, Croatia, is a musical instrument that creates its musical notes through the action of sea waves on a set of pipes that are located underneath the steps shown in the picture. The sound from the pipes comes out through the regular slits in the vertical part of the top step.
Example 3 continued

(a) Calculate the length, $L$, of an organ pipe, with one closed end, that produces a fundamental standing wave of wavelength 2.60 m.

$$L = \frac{1}{4} \lambda = \frac{1}{4} \times 2.6 = 0.650 \text{ m}$$
Example 3 continued

The Sea Organ contains organ pipes of several different lengths. Explain why the differences in length of the organ pipes affect the sounds that are heard.

- The length of the pipe is proportional to the wavelength of the sound wave produced.
- As the frequency (pitch) of the sound produced depends inversely on its wavelength \( v = \frac{f}{\lambda} \), the longer the pipe the lower the frequency.
Example 3 continued

• (d) The speed of sound in cold air is slower than it is in warm air.
• Calculate the difference between the 3rd harmonic frequency (1st overtone) heard in summer (35°C), and the 3rd harmonic frequency heard in winter (–2°C).
• Speed of sound in air at 35°C = 353 m s⁻¹
• Speed of sound in air at –2°C = 330 m s⁻¹

\[ \lambda = \frac{4L}{3} = \frac{4 \times 0.65}{3} = 0.8666 \text{ m} \]

\[ f_{\text{cold}} = \frac{330}{0.8666} = 380.77 \text{ Hz} \]

\[ f_{\text{hot}} = \frac{353}{0.8666} = 407.31 \text{ Hz} \]

Difference = 407.31 – 380.77 = 26.5 Hz